Helicopter Down-Wash Speeds and Profile

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This short paper describes the vertical velocity profile of the down wash below the main rotor of a helicopter. The starting point is from:

`https://en.wikipedia.org/wiki/Disk_loading`

which concerns "disk loading" of an airscrew. Note that my choice of variable names is slightly different than that in the wiki article.

The "rotor disk" referred to in this article is an imaginary circular disk that is swept out by the turning of the rotor blades of a helicopter. The diameter, D, is twice the length, r, of one rotor blade. The disk area, A, is

$$A = \pi r^2 = \pi \frac{D^2}{4} \tag{1}$$

We start by defining that the air *above* the main rotor is at rest. That is, $v_0 = 0$. We also define two other air movement speeds: v_1 is the speed of the air mass just after it has cleared the main rotor, i.e., just below the rotor disk; and v_f is the speed of the down wash air stream *far* below the rotor disk. What *far* means will become clear by the end of the paper.

From the wiki article section, "Momentum theory", we have the equation for the mass flow rate $\dot{m}$ through the disk

$$\dot{m} = \frac{dm}{dt} = \rho A v_1 \tag{2}$$

where ρ is the density of the air (sea level value, $\rho = 1.2041\text{kg/m}^3$). We will also use the fact that the upward thrust, T, resulting from "recoil" due to the acceleration of the down wash air stream, is the rate of change of the total momentum of the down wash air stream through the disk (i.e., momentum is conserved)

$$T = \dot{m} v_f = \rho A v_1 v_f \tag{3}$$

Note that, in a hover or in straight and level flight, the upward thrust must equal the gravitational force pulling the helicopter down. That is

$$T = Mg \tag{4}$$

where M is the mass of the helicopter and g is the standard earth surface gravitational acceleration (9.8m/s^2). Also from the same section of the wiki article, the total power expended by the rotor must equal the rate of change in the total kinetic energy of the accelerated down wash air stream,

$$T v_1 = \frac{1}{2} \dot{m} v_f^2 \tag{5}$$

Substituting $\dot{m}v_f$ (equation 3) for T

$$\dot{m}v_f v_1 = \frac{1}{2}\,\dot{m}v_f^2 \tag{6}$$

makes it obvious that

$$v_f = 2\,v_1 \tag{7}$$

We need to continue, as we want to know the actual down wash air stream velocity profile. Using equation 2 to substitute for $\dot{m}$ and equations 4 and 7 to make substitutions in equation 5 gives us

$$Mg = \frac{1}{2}\,\pi\rho D^2 v_1^2 \tag{8}$$

Solving equation 8 for v_1 gives the down wash speed just below the rotor disk plane:

$$\boxed{v_1 = \sqrt{\frac{2Mg}{\pi\rho D^2}}} \tag{9}$$

We can see that v_1 decreases as the rotor diameter increases and that v_1 increases as the weight of the helicopter increases.

As an example, take the Bell 206B3 JetRanger mentioned in the wiki reference. The gross mass of this helicopter is 1451kg and the total disk area is stated to be 81.1m^2. This means the disk diameter is 10.16m. Using equation 9 and the above mentioned values for ρ and g, we find that the initial down wash speed is 8.53m/s (16.6knots). From equation 7, we can also see that the speed far below the rotor is about 17m/s (33knots).

It may seem non-intuitive that the stream speed should increase as the air gets farther below the rotor. The explanation is that as the air goes through the rotor disk and is accelerated down, it is also compressed. This compression represents additional energy that is transferred to the air. As the compressed air is not contained, it decompresses and expands. The air can expand horizontally out from the stream, or downward in the stream, but cannot expand upward because of the column of higher pressure compressed air above it. The downward part of the expansion increases the downward speed of the stream. This expansion is essentially iso-thermal and air is close to an ideal gas. The rate of expansion is proportional to the ratio of the pressure in the compressed air to that of the air surrounding it. Thus,

$$\frac{dP}{dt} = -\alpha(\frac{P}{P_0} - 1) \tag{10}$$

where α is the proportionality constant. This number is a function of thermodynamic properties of air and the diameter of the rotor. Equation 10 has the exponential solution

$$P = (P_i - P_0)e^{-\frac{\alpha}{P_0}t} + P_0 \tag{11}$$

where P_i is the initial pressure just below the rotor.[1]

As the pressure decays, the rate of expansion decays and the acceleration due to the expansion subsides. We know, from the total power balance, that $v_f = 2v_1$, so the speed profile as a function of the distance, z, down from the rotor disk is

$$\begin{aligned} v(z) &= v_1 + v_1(1 - e^{-\kappa z}) \\ &= v_1(2 - e^{-\kappa z}) \end{aligned} \tag{12}$$

where κ is related to α and P_0. We can see from the graph in Figure 1 that the value of κ determines how quickly $v(z)$ approaches v_f. For realistic values of κ, $v(z)$ closely approaches v_f by one rotor diameter below the rotor disk.

While the down wash speed profile resulting from a helicopter rotor is not as intuitive as some might initially think, it is a straight forward task to compute both the initial value and the value far down stream. The profile of speeds between these two values is exponential, growing rapidly just below the rotor disk and asymptotically approaching the final value by about one rotor diameter below the rotor.

[1] Proof: $\frac{dP}{dt} = -\frac{\alpha}{P_0}(P_i - P_0)e^{-\frac{\alpha}{P_0}t} = -\alpha\left(\frac{((P_i - P_0)e^{-\frac{\alpha}{P_0}t} + P_0 - P_0}{P_0}\right) = -\alpha\left(\frac{P}{P_0} - 1\right)$

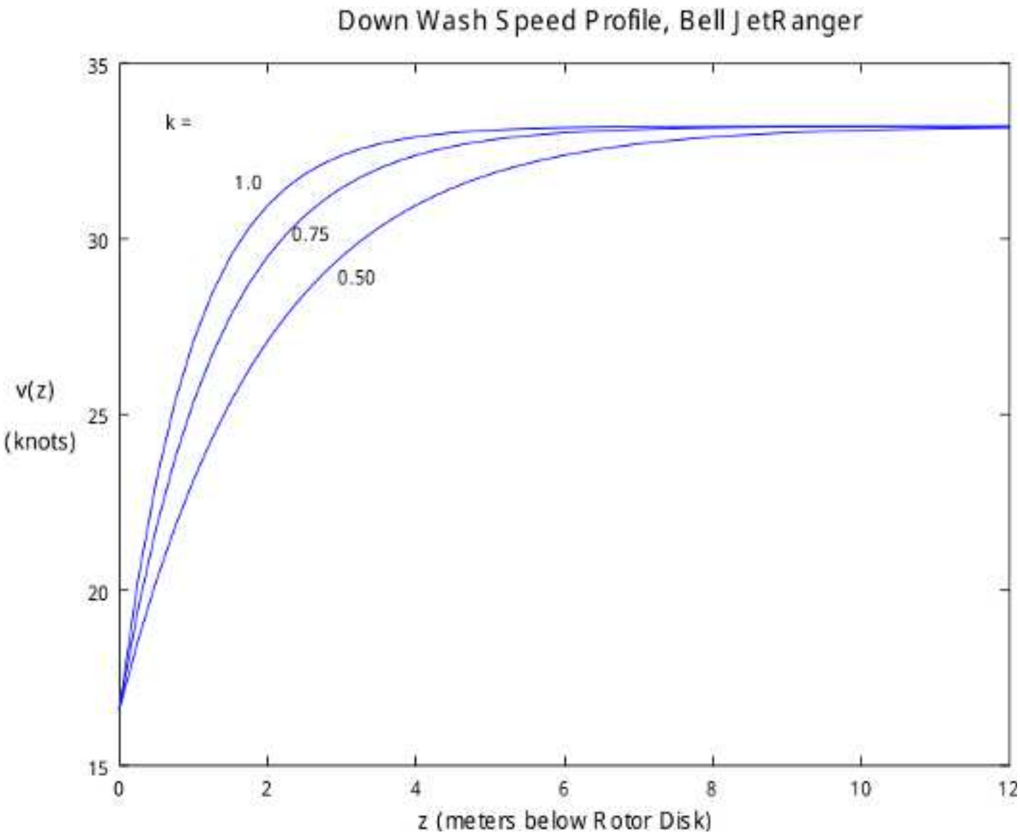

Figure 1: Down Wash Speed Profile for Bell JetRanger ($D \simeq 10.2$m)